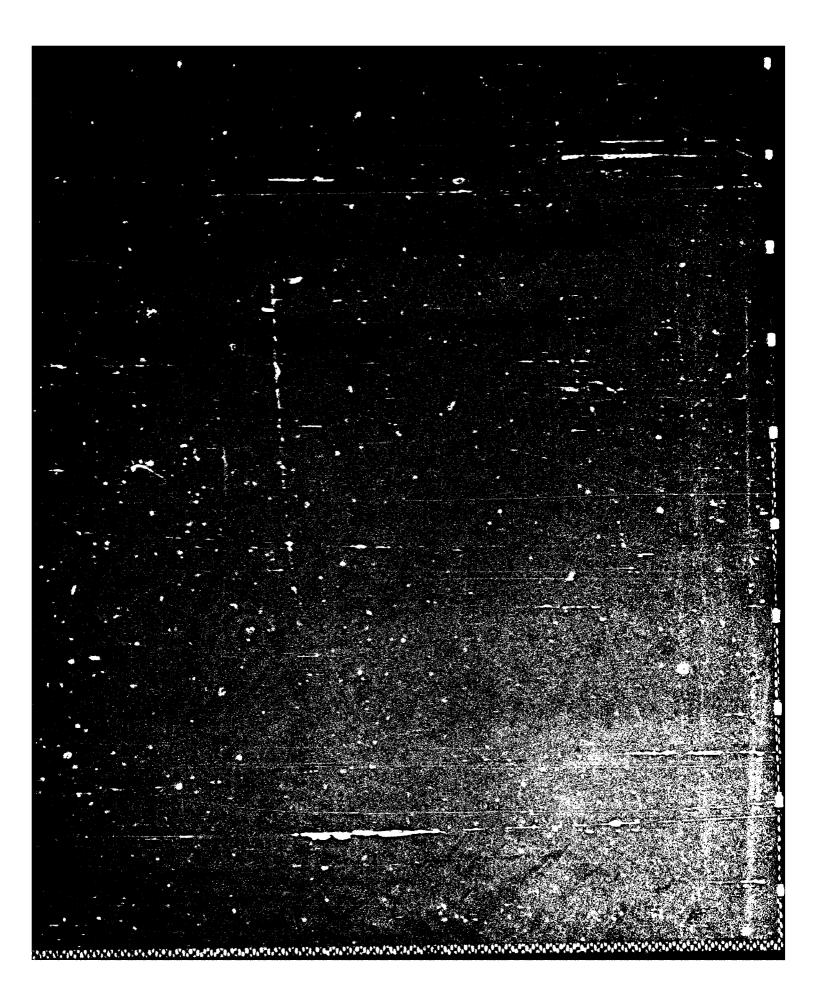
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#### I. INTRODUCTION

A serious problem in the theory of the turbulent flow of two-phase systems is that while very often one of the phases is a bona fide fluid, the other one, if isolated, would have a collision mean free path so large that the usual continuum approximation cannot be applied to its description. Examples of such a situation include flows in pulverized coal gasifiers, certain problems occurring in meteorology, such as flows in clouds and many others. In this paper we suggest a description of such flows by means of a mixed system of equations: one describes the flow of the possibly dilute (generally particulate) phase by means of a transport equation of the Boltzmann type, while using the Navier-Stokes (NS) equations for the fluid phase. Generally, however, one cannot use either the Boltzmann equation or NS in their "textbook form". First of all, the interaction between the phases is a dissipative one; hence the volume element of the phase space is not constant in time and the Boltzmann equation has to be appropriately modified.

Second, we use a statistical description of the turbulent flow of the fluid phase  $^1$  and thus, smooth solutions of NS are used only to describe the average flow whereas the probability distribution of velocity fluctuations is given by an expression derived in a previous paper of ours.  $^2$ 

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The paper is organized as follows. In the next section we derive the modifications necessary to transport equations of the Boltzmann type in the presence of dissipative forces. We derive an explicit expression for the extra term appearing in the transport equation in terms of the force acting upon a single particle. In Sec. 3 we briefly review the formalism developed in Ref. 2 and apply it to the coupled two-phase system. In particular, we derive moment equations for the particle phase in the leading approximation of a Chapman-Enskog expansion. The results are discussed in Sec. 4; in particular, we point out possible directions in which our results can be generalized.

#### 11. PARTICLE TRANSPORT IN A DISSIPATIVE MEDIUM

Consider a system of particles with an ensemble average of the phase space density given by  $f(x^i, v^i)$ , where  $x^i$  and  $v^i$  stand for the components of the particle position and velocity vector, respectively. The integral of f over the entire phase space gives the total number of particles. The particles are acted upon by "slowly varying" forces exerted by the carrying medium: the interaction between the particles

C. DeDominicis and L. Peliti, "Field Theory, Renormalization and Critical Dynamics Above T<sub>c</sub>: Helium, Antiferromagnets and Liquid-Gas Systems," <u>Phys. Rev.</u>, Vol. B18, pp. 353-376, 1978, and references quoted there.

<sup>&</sup>lt;sup>2</sup>G. Domokos, S. Kovesi-Domokos and C. K. Zoltani, "Random Systems, Turbulence and Disordering Fields," <u>Phys. Rev. A</u>, submitted for publication.

is described by means of a collision functional. In the Boltzmann approximation to the transport problem, cf., e.g., Huang<sup>3</sup>, Chapter 5, the change in time of the phase space density is given by the equation:

$$\frac{d}{dt}(fd^3xd^3v) = F[f]d^3xd^3v , \qquad (2.1)$$

where F stands for the collision functional and the time derivative is the total derivative. In this paper we assume for the sake of simplicity that the phases are non-reactive. Hence, in particular, the particle mass (or, in an inhomogeneous particle phase, the average particle mass), m, stays constant. In the present work, we need not specify a detailed form of the collision functional, F[f]. In general, its precise form appears to be of little immediate relevance, as long as the average momentum transfer in a binary collision is nonzero. The "textbook form" of the Boltzmann equation is arrived at by using the constancy in time of the volume element in phase space (Liouville's theorem) and dividing out by the phase space volume element. In the problem we are interested in, Liouville's theorem is not valid: the force by means of which a carrying fluid acts upon the particles is a velocity dependent one. Let us assume that the equations of motion are of the form:

$$\frac{dx^{i}}{dt} = v^{i},$$

$$\frac{dv^{i}}{dt} = (\frac{1}{m})f^{i}.$$
(2.2)

With this, one readily finds:

$$\frac{d}{dt}(d^3xd^3v) = (\frac{\partial f^i}{\partial v^i})/m . \qquad (2.3)$$

Hence, the correct form of the transport equation describing the time change of the phase space density becomes:

$$\frac{\partial f}{\partial t} + v^{i} \frac{\partial f}{\partial x^{i}} + (\frac{f^{i}}{m}) \frac{\partial f}{\partial y^{i}} + \frac{f}{m} (\frac{\partial f^{i}}{\partial y^{i}}) = F[f]. \tag{2.4}$$

If all forces were conservative, the last term on the l.h.s. of Eq. (2.4) would vanish.

We model the interphase interaction as follows. Assuming that the particle phase consists of particles which have random shapes and are randomly oriented, it is clear from Galilean invariance that the force which acts upon a particle of a given velocity must be proportional to the relative velocity of the particle and the surrounding fluid element

<sup>&</sup>lt;sup>3</sup>K. Huang, <u>Statistical Mechanics</u>, Wiley, New York, 1978.

We write the force in the form:

$$f^{i} = 6\pi \nu a \rho_{f}(u^{i} - v^{i})D(R_{p})$$
, (2.5)

where the components of the fluid velocity are denoted by  $\mathbf{u}^i$ ,  $\rho_f$  stands for the mass density of the fluid,  $\nu$  is an effective kinematic viscosity acting between the fluid and the particle. The dimensionless function  $\mathrm{D}(\mathsf{R}_p)$  describes the deviation of the force from Stokes' law. It depends on the dimensionless "particle Reynolds number",  $\mathrm{R}_p = \left|\mathbf{u}^i - \mathbf{v}^i\right|_{\overline{\nu}}^a$ , where

on the dimensionless "particle Reynolds number",  $R_p = |u^2 - v^2| \frac{\omega}{\nu}$ , where a is the average radius of a particle. The normalization has been chosen in such a way that D(0) = 1. At this stage we do not specify the function D any further. For high values of  $R_p$ , there exist no good

theoretical expressions for it; probably, one has to be satisfied with a semiempirical relation. Corresponding to a force given by Eq. (2.5), there is a force acting upon the fluid, its density being given by:

$$y^{i} = -\int d^{3}v f^{i}$$
 (2.6)

Using Eq. (2.5), a straightforward computation leads to the expression of the divergence appearing in the transport Eq. (2.4). We find:

$$\frac{\partial f^{i}}{\partial v^{i}} = -6\pi \nu a \rho_{f}(3D + R_{p}D') , \qquad (2.7)$$

where the prime denotes the derivative of D with respect its argument. We see that even if the drag is assumed to be obeying Stokes' law, i.e.,  $R_{p}$  is small, there is a finite correction to the transport equation arising from the shrinkage of the phase space.

A further remark is in order at this point. As discussed in the Introduction, we are proposing to apply a statistical theory of the turbulent flow of the fluid phase. As a consequence, the components,  $\mathbf{u}^i$  of the fluid velocity are to be treated as random variables. Thus, the transport equation has to be averaged over the probability distribution of  $\mathbf{u}^i$ .

#### III. STATISTICAL THEORY. MOMENT EQUATIONS

We start by reviewing the statistical theory of a chaotic process (turbulence in particular) as formulated in Ref. 2. We need a formulation more general than the one just suitable for the investigation of a single phase system.

Consider a dynamical system which exhibits chaotic behavior. We denote the collection of dynamical variables describing the system by X. In practically important cases X is an element of a vector bundle over space-time, i.e.,  $\mathbb{R}^3 \times \mathbb{R}^1$ . We assume that the dynamical equations satisfied by X are of the form,

$$\partial_{+} X - \Psi[X] = 0 , \qquad (3.1)$$

where  $\Psi$  is a map of the fibration onto itself. Following the ideas of Martin <u>et al.</u>, and DeDominicis <u>et al.</u>, Ref. 1, it was found in Ref. 2 that the probability measure over the (generally infinite dimensional) phase space characterizing chaotic behavior is governed by a probability measure proportional to:

DX Det
$$\left(\partial_{t} - \frac{\delta \Psi}{\delta X}\right) \exp - \left\langle \partial_{t} X - \Psi \right| \partial_{t} - \Psi \rangle$$
, (3.2)

where DX stands for a volume element over the bundle and < | > is a scalar product over it. The determinant has to be given an appropriate meaning, typically in terms of an integral over Grassmann variables.

Our coupled system of equations, consisting of Eqs. (2.4) and (2.5) and NS with an external force density given by Eq. (2.6), does not fit directly into this formalism. However, a Chapman-Enskog expansion of the transport equation (cf., Huang, loc. cit.) leads to a system of equations which is of the form Eq. (3.1); moreover, it also provides a practical approach to the solution. In this work, we restrict ourselves to the leading approximation of the Chapman-Enskog expansion. To that order (local equilibrium), the physics is entirely governed by the conservation laws of particle number, energy and momentum. Due to the fact that the transport equation is not entirely of the standard form, we also have to rederive the conservation laws.

Let us introduce some notation. The number density of particles is denoted by n, and if A is any function on the phase space, its velocity average is denoted by angular brackets:

$$n = \int d^3v f$$
,  $\langle A \rangle = [\int d^3v f A]/n$ . (3.3)

Let Y denote any quantity conserved in the interparticle collision, i.e.,

$$\int d^3v \ Y \ F[f] = 0,$$
 (3.4)

cf., Huang, loc. cit. Then, in view of eqs. (2.4) and (3.4) we have

$$\int d^3 v \ Y \left\{ \frac{\partial}{\partial t} + v^i \frac{\partial}{\partial x^i} + (\frac{f^i}{m}) \frac{\partial}{\partial v^i} + (\frac{\partial f^i}{\partial v^i})/m \right\} f = 0.$$
 (3.5)

On performing some integrations by parts in order to bring the derivatives over to Y, we obtain the conservation law in the form:

$$\frac{\partial}{\partial t}(n < Y>) + \frac{\partial}{\partial x}i(n < v^{i}Y>) - n < v^{i}\frac{\partial Y}{\partial x^{i}}> - (\frac{n}{m}) < f^{i}\frac{\partial Y}{\partial v^{i}}> = 0.$$
 (3.6)

<sup>&</sup>lt;sup>4</sup>P. Ramond, <u>Field Theory. A Modern Primer</u>, Benjamin, New York, 1981, Chapter VII.

The remarkable fact to be noticed is that, although, in the course of integrating by parts the third term of Eq. (3.5), one would pick up a term proportional to the derivative of  $f^i$  with respect to the velocity, that term cancels exactly with the last term. Thus, the expression of the consevation laws is unchanged whether or not the force is velocity dependent. (The standard textbook derivations of the Boltzmann equation and the equations of hydrodynamics therefrom, typically assume that the forces are conservative.) It is almost needless to remark, however, that beyond the leading order of the Chapman-Enskog expansion, the expression of the pressure tensor will be modified due to the velocity dependence of the forces.

We are now ready to cast our equations into a form given by Eq. (3.1). In the approximation we are dealing with, and on denoting,  $:v^i>\equiv w^i$ , the dynamical equations are:

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$$m \frac{Dnw^{i}}{Dt} + \delta^{i}Q - n \langle f^{i} \rangle = 0 , \qquad (3.7a)$$

$$(\partial_t + u^r \partial_r) u^i + \rho_f^{-1} \partial^i p - \nu \nabla^2 u^i + (\frac{n}{\rho_f}) \langle f^i \rangle = 0.$$
 (3.7b)

(The somewhat asymmetric appearance of the coupling term between the two equations in Eq. (3.7) is due to the fact that originally we defined the interphase coupling in terms of a force acting upon a single particle.) In these equations,  $\partial_i \equiv \partial/\partial x^i$  an Q stands for the pressure exerted by the particulate phase upon itself. In most cases, unless the particle density is very high, one may set Q = 0, and we shall do so in what follows. The symbol D/Dt stands for the Galilei invariant time derivative, D/Dt  $\equiv \partial_t + w^T \partial_r$ ; on writing down the NS equation for the fluid phase, we implicitly assumed incompressibility. This cannot be done for the particle phase: the equation of continuity has to be taken into account as a constraint.

In order to take the equation of continuity into account, we notice that the equation of continuity for the particle phase reads as follows:

$$\partial_t n + \partial_i (nv^i) = 0 . (3.8)$$

In the statistical theory, we will have to integrate over n and  $v^i$  with a volume element proportional to Dn D $v^i$ . In the standard fashion, (cf. Ramond, <u>loc. cit.</u>) this volume element will be replaced by

$$D\sigma Dr Dn Dv^{i} \delta[\partial_{t} n + (\partial_{r} v^{r}) n + v^{r} \partial_{r} n] \exp \int d^{3}x dt L_{g}, \qquad (3.9)$$

where  $\sigma$  and  $\tau$  are Fadeev Popov ghost fields, hence, they are Grassmann algebra valued. The ghost Lagrangian, L  $_{\rm g}$ , is given by the functional

derivative of the continuity equation with respect to n, sandwiched between the ghost fields:

$$L_{g} = \sigma(\partial_{t} + (\partial_{r}v^{r}) + v^{r}\partial_{r})\tau . \qquad (3.10)$$

The presence of a delta-functional,  $\delta[\ldots]$ , in the volume element Eq. (3.9), makes computations rather awkward. The standard trick familiar from gauge theories, namely a Gaussian averaging over a family of gauge conditions, is not applicable here, because Eq. (3.8) is not a gauge condition. However, one may replace the delta-functional by a Gaussian,  $\underline{viz}$ .

$$\delta[\partial_t n + \partial_r (nv^r)] \approx \exp - \lambda^{-2} \int d^3x dt \left[\partial_t n + \partial_r (nv^r)\right]^2$$
, (3.11)

where  $\lambda$  is a parameter, to be let go to zero at the end of the calculation. Furthermore, the Gaussian functional in Eq. (3.11) needs not be normalized, since the physically interesting quantities are expressed in the form of cumulants: their calculation is not affected by field-independent factors in the probability measure. Combining now these results, we obtain the constrained functional volume element, DP, for the stochastic (infinite dimensional Wiener) integration over the dynamical variables of the particle phase:

$$D\Phi = D\sigma D\tau Dv^{i} Dn \exp T , \qquad (3.12)$$

where

$$\Gamma = \int d^3x dt \left\{ -\lambda^{-2} \left[ \partial_t n + \partial_r (nv^r) \right]^2 + \sigma \left[ \partial_t + v^r \partial_r + \partial_r v^r \right] \tau \right\} . \tag{3.13}$$

Given the fact that the fluid phase has been treated as an incompressible one, the equation of continuity needs not be taken into account by means of the Fadeev-Popov procedure. All one needs is to project out the transverse component of the hydrodynamical velocity field,  $\mathbf{w}^i$ , which can be done in a well-known way.

In order to complete the system Eq. (3.7), one has to compute the quantity  $\langle f^i \rangle$ . Given a specific model for the drag function in Eq. (2.5), this may or may not be computed in a closed form; nevertheless, the requirement of Galilean invariance tells us that the final expression can again be cast in a form analogous to Eq. (2.5), with the hydrodynamic velocity,  $\mathbf{w}^i$ , replacing the particle velocity. (Of course, the explicit expression of a dimensionless drag function in terms of  $\begin{vmatrix} \mathbf{u}^i & \mathbf{w}^i \end{vmatrix}$  will be different from the originally assumed form occurring in Eq. (2.5).)

Following now the general procedure outlined in Ref. 2, we have to construct a probability measure in the six-dimensional vector space formed by the components of  $\vec{u}$  and  $\vec{w}$ . Given the fact that the system Eq. (3.7) consists of a set of coupled equations, a consistent procedure is to explicitly construct the functional  $\Psi$  in Eq. (3.1). To this end,

we define the components of the vector  $X^A$  (1  $\leq$  A  $\leq$  6) as follows.

First, we define:

$$X^{A} = u^{A}$$
  $(1 \le A \le 3)$ ,  $X^{A} = w^{A-3}$   $(3 \le A \le 6)$ .  $(3.14)$ 

In a similar fashion, we define embeddings of the pressure gradient and the viscous damping terms into the six dimensional velocity space. This is straightforward: we define a diagonal tensor projecting onto the fluid phase:  $P_{AB} = \delta_{AB}$  if  $1 \le A$ ,  $B \le 3$ ,  $P_{AB} = 0$  otherwise. With this, we define:

$$V_{AB} = \rho_f \nu \nabla^2 P_{AB}$$
, 
$$P^A = \partial_A p \quad (1 \le A \le 3), \quad P^A = 0 \text{ otherwise}. \tag{3.15}$$

In order to proceed further, we have to define a constant tensor, M, with components:

$$M_{AB} = \delta_{A'B'3}$$
 (A < B),  $M_{AB} = \delta_{A-3'B}$  (A > B),  $M_{AB} = 0$  otherwise. (3.16)

In terms of these quantities, we can define the six-vector of relative velocity,  $\mathbf{Y}^{\mathbf{A}}$ , as follows:

$$Y^{A} = X^{A} - M_{B}^{A} X^{B} . ag{3.17}$$

One readily verifies,  $Y^A Y_A = 2(u^i - v^i)(u_i - v_i)$ . Thus, the particle Reynolds number can be written in terms of six-vectors as  $R_p = (a/2^{1/2} \nu) |Y|$ . The next step is to express the interphase coupling term; this is done by constructing the vector:

$$d^{A} = -\langle f^{A} \rangle - (1 \leq A \leq 3), \quad d^{A} = \langle f^{A \cdot 3} \rangle - (4 \leq A \leq 6). \quad (3.18)$$

Finally, the embedding of the gradient operator into the six dimensional space is given by:

$$D_{A} - \partial_{A} = (1 \le A \le 3), \quad D_{A} = \partial_{A-3} = (4 \le A \le 6).$$
 (3.19)

With these quantities, the system Eq. (3.7) can be written as follows:

$$\{\partial_{t} \delta_{B}^{A} + X^{A} D_{B}^{-} + 2P_{R}^{A} X^{R} D_{S}^{-} P_{B}^{A} + -P_{R}^{A} X^{R} D_{B}^{-} - X^{A} D_{S}^{-} P_{B}^{S} \} \{\delta_{C}^{B} nm - (\rho_{f}^{-} - nm) P_{C}^{B} \} X^{C}^{-} + P^{A}^{-} + V_{C}^{A} X^{C}^{-} + d^{A}^{-} = 0 .$$

$$(3.20)$$

This form allows us to read off the form of the functional  $\Psi$  and thus constuct the probability measure Eq. (3.2). The volume element is the product, D $\!\Phi$  D $\!w^i$ .

#### IV. DISCUSSION

Our main purpose in this work has been to establish a formalism for the dynamical description of the turbulent flow of a two-phase system in which the particle phase is of an arbitrary concentration. The application of the Chapman-Enskog expansion to the Boltzmann equation, on the one hand, provides a powerful practical approach to the solution, on the other hand, at any order of the expansion it casts the equations into a form which can be handled by means of standard techniques of quantum field theory, including resummation techniques of perturbation expansions such as the use of the renormalization group, etc. In order to apply the present formalism to realistic cases, such as flows in a pipe, one has to find an average flow around which one carries out a Reynolds decomposition of the fields. If one restricts oneself to the leading order of the Chapman-Enskog expansion, and one considers some simple types of average flows, finding an average flow is not a very hard task. For instance, in the practically important case of a stationary flow in a pipe under uniform pressure gradient, it is easy to show that  $\mathbf{u}^{i}$  =  $\mathbf{w}^{i}$ (i.e.  $Y^A = 0$  and, hence,  $d^A = 0$ ) is a suitable solution around which such a decomposition can be carried out.

There is no mathematically sound way of deciding how good the Chapman-Enskog expansion is in any given situation. However, intuitively one expects that even though a dilute particle phase, if left in isolation, would be quite far from local equilibrium, an interaction with the fluid phase will bring the combined system quite close to local equilibrium and thus, justify the use of a truncated Chapman-Enskog expansion. To some extent, this would justify the two-fluid description of the two phase flow; see also Drew for a discussion. It is to be emphasized, however, that our approach is not tied to the use of such an expansion.

As a final remark, we wish to draw attention to the amusing fact, expressed by Eq. (3.6), namely that the form of the conservation laws is unchanged despite the fact that the velocity dependence of the forces invalidates Liouville's theorem. To some extent, this situation is similar to the one expressed by the so-called "Hilbert paradox" in transport theory: the Boltzmann distribution function contains more information than the totality of its few moments, see e.g., Uhlenbeck.

<sup>5</sup> D. A. Drew, "Mathematical Modelling of Two-Phase Flow," in <u>Annual</u> <u>Review of Fluid Mechanics</u>, Vol. 15, p. 261. Annual Reviews, Palo Alto, 1963.

<sup>&</sup>lt;sup>6</sup>G. E. Uhlenbeck, "The Validity and the Limitations of the Boltzmann Equation," <u>Acta Phys. Austriaca</u>, Supp. X, pp. 107-110, 1973.

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